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“The Modal Interpretation of the Principle of Least Action and Feynman Path Integral”

Four questions

To understand QM, we have to answer the questions:

1. Do the **possible states and histories exist** before their observations?

2. Why do the **probabilities of possible states depend** on the knowledge and the selection of the observer?

3. How is the **quantum probability related** to the classical causality?

4. How does the **classical reality** (objects and laws) **arise** from the quantum reality?

One of the ways to answer is to combine the modal and the information approaches to QM.

Outline

1. **The Principle of Least Action (PLA) is a classical limit of Feynman Path Integral.**

2. **Leibniz could "explain" the meaning of the action. The modal interpretation of the PLA.**

3. **The action is related to redundancy of information in the environment (Quantum Darwinism).**

Part I

The Principle of Least Action and Feynman Path Integral

Possibilities

The notions *possible, virtual, or imaginary* states or histories are applied:

- in the variational principles
- in the interpretations of quantum mechanics
- in Feynman path integral
- in quantum cosmology

It is accepted that all such notions are merely formal mathematical tools for calculation.

Why are the models with possible states or histories so much successful?

The principle of least action (PLA)

***A. Einstein*:** the whole general relativity can be developed based on PLA

M. Planck: PLA dominates above all reversible phenomena of physics (including the Laws of Conservation of Energy and Momentum)

A. Eddington: there are two great generalizations of nature, PLA and the Second Law of Thermodynamics

R. Feynman: the relation between symmetry laws and conservation laws is connected with the PLA "because they come from quantum mechanics" (*Character of physical law*)

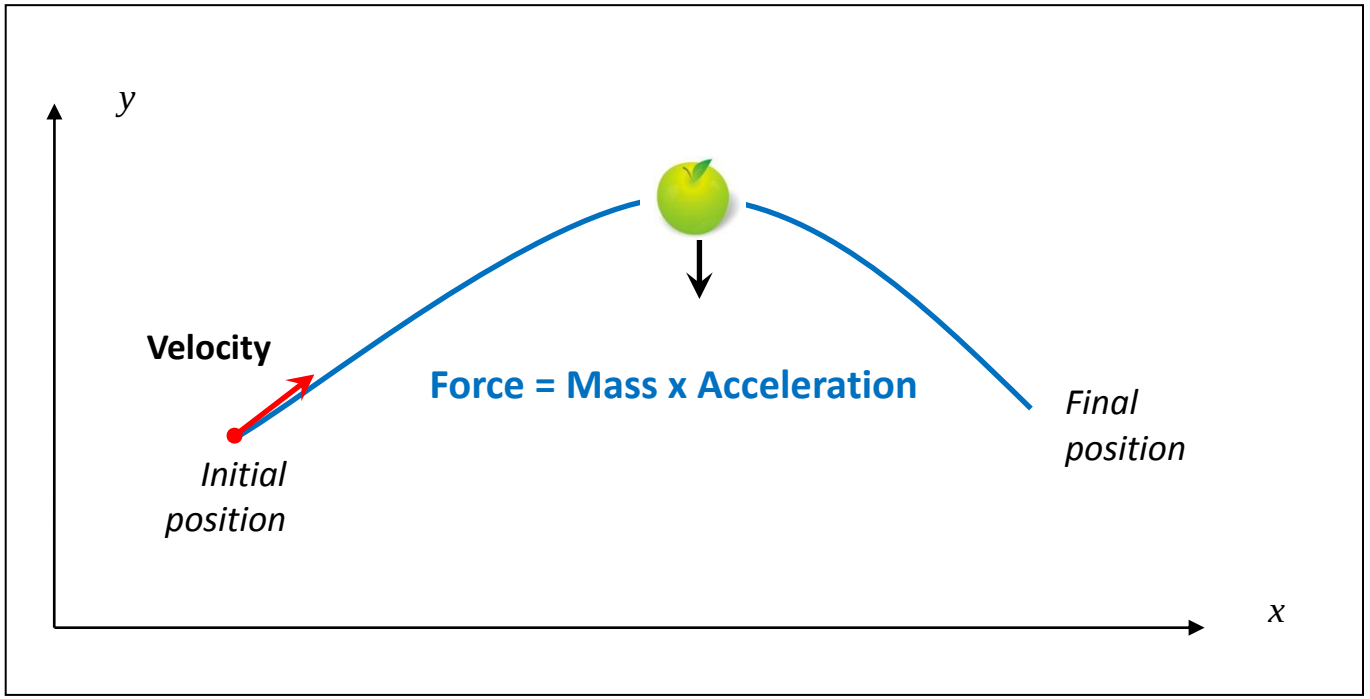
T. Moore: PLA lies at the core of much of contemporary theoretical physics (*Macmillan Encyclopedia of Physics, 1996, Vol. 2, p. 840*)

The philosophical issues of the PLA.

1. **How does a physical system "choose" an actual path from all possible paths? Why does it have a minimal action?**

2. **Whether the possible histories in the PLA have some grade of reality or not? What if they take place in the "space" of possible events?**

The flying apple - 1

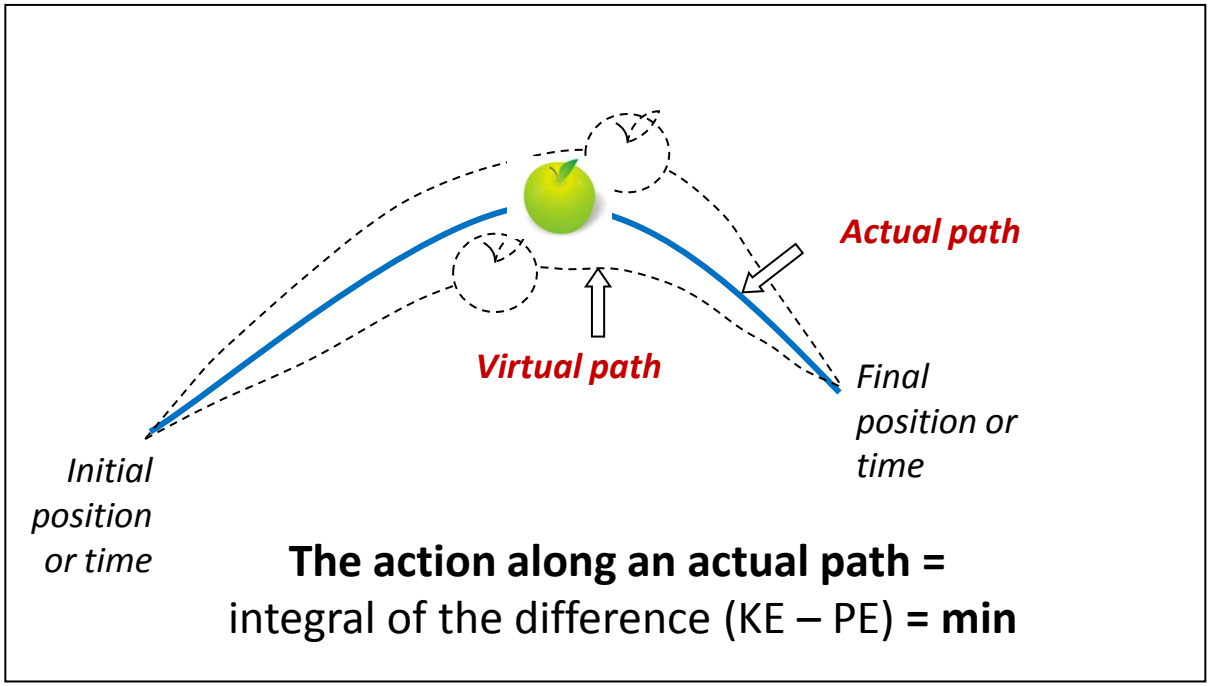


Newtonian mechanics

An apple possess an **internal tendency** to continue in motion with the same velocity along a straight line (inertia).

When the apple **"perceives"** at a distance the effect of the force, it changes its own the velocity.

The flying apple - 2



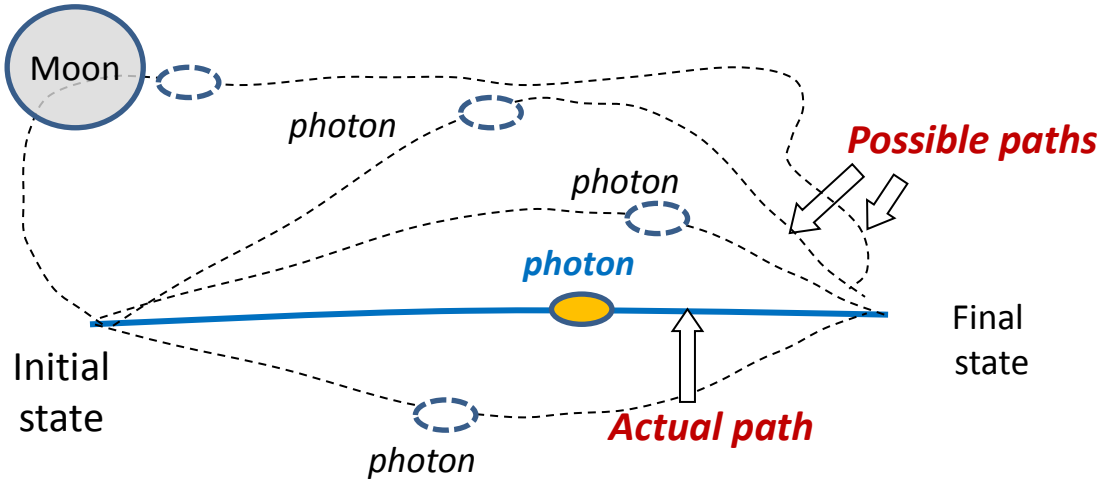
Lagrangian mechanics

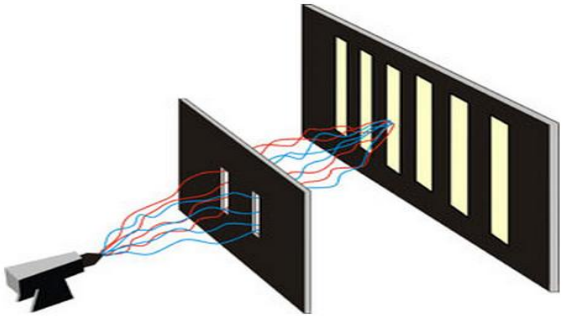
The Principle of Least or Stationary Action

We do not need to know how the apple works its way from one event to another; we need to know **only the initial and the final positions** of the apple **and times**.

Along the actual path, the **action reaches a minimum** compared with all possible paths.

Feynman's path integral method





$$p = |K|^2 = \max$$

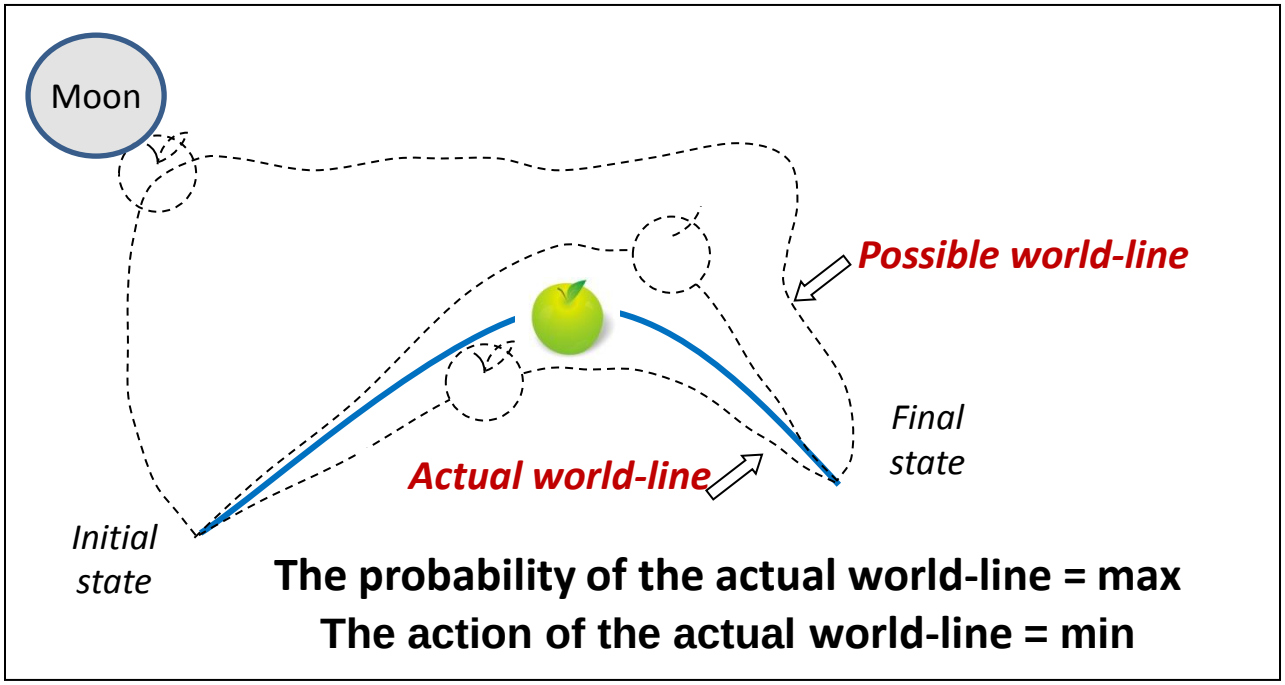
The probability of the actual path = max

$$K(b,a) = \sum_{\substack{\text{all possible} \\ \text{path} \\ \text{from a to b}}} \varphi, \quad \varphi = const \cdot e^{iS/\hbar}$$

The photon **simultaneously** takes an infinite set of all possible alternative histories in superposition (on the Moon too).

Each possible histories has a **quantum phase corresponded to the action**. They interfere. The actual history is the limit of a narrow bundle of the possible **histories significantly contributing to the quantum amplitude**.

The flying apple - 3



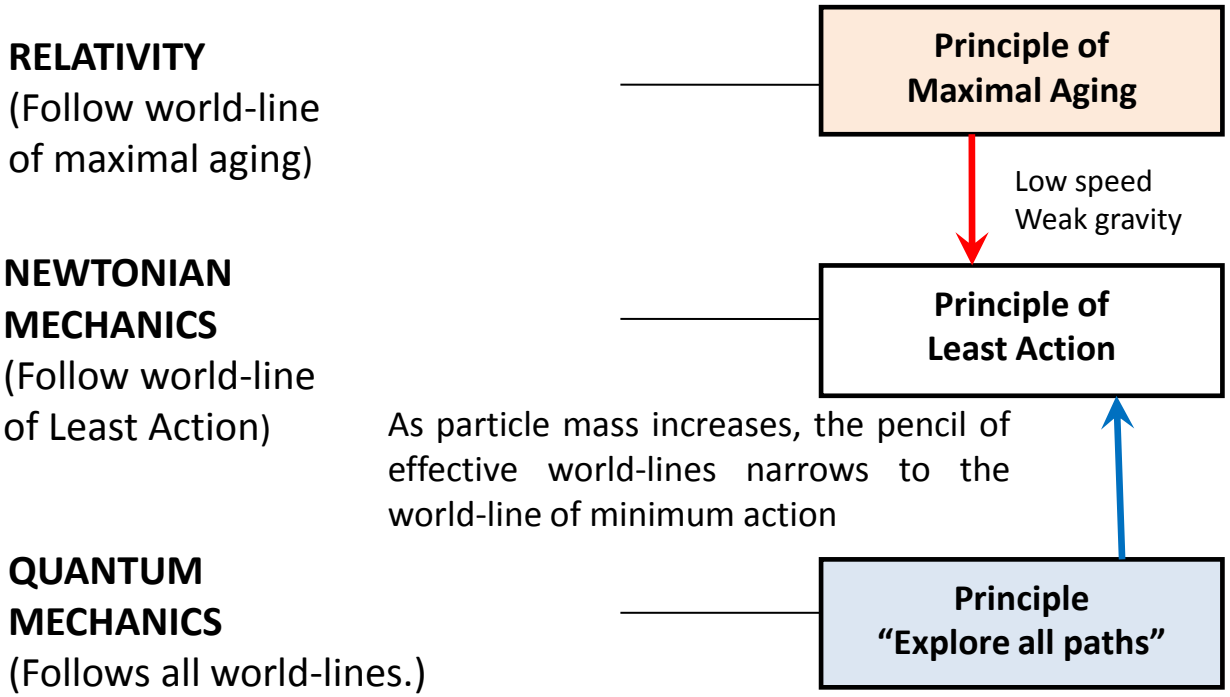
Quantum mechanics

Feynman's path integral method

The apple, as well as a photon **simultaneously** takes an infinite set of all possible world-lines between initial and final states (on the Moon too).

We observe the only path with the **maximal probability** and **minimal action**. It is the geodesic line.

The unification of the descriptions



Nature's command for the apple:

- Follow the path of maximal aging (or maximal proper time)!
- Follow the path of least action!
- Explore all paths!

(Taylor E.F. A call to action. American Journal of Physics. 71 (5), 2003)

Part II

The Modal Interpretation of the Principle of Least Action

The modal interpretation of the PLA

1. There is a **deep ontological relationship** between the possible paths in the PLA and possible quantum histories in the FPI.

2. Two modalities of being. The possible events and histories "occur" **simultaneously in the possible realm of our world**. The actual events and histories occur in the actual realm of the world.

3. It replaces the classical representation of the motion along a single history by the simultaneous motions along **an infinite set of all possible histories**.

Let us examine the possible paths in the PLA from the perspective of Leibniz's metaphysics.

Leibniz's doctrine of the striving possibilities

"*All possible things have the essence and tend towards existence*".

A physical system tends towards existence in all possible ways or moves from each initial actual event **along all possible histories.**

These possible movements occur **simultaneously in the possible modality.**

The possible histories are **mutually exclusive** or **not compossible** (not compatible) in four-dimensional **space-time**. But in the possible modality, there is no space-time yet.

"*Everything that expresses essence or possible reality, strive with equal right for existence in proportion to the amount of essence*" (G. W. Leibniz. Philosophical Essays. 1989. p. 150).

The Action and the Essence

Let us accept that the actions of the possible histories somehow correlate with the essence.

A certain kind of a **"collision"** or **"competition"** occurs between the possible histories. The result of such a "competition" has the **maximal essence** and is manifested in the actual history.

Leibniz: *from the collision of all possibilities, only those things will be actualized that contain* ***the greatest number of the possibilities****. So the world arises in which the largest part of the possible things is actualized* (Ibid. p. 155).

Leibniz's example: a **straight line** among all lines, a **circle** among all figures as the most capacious ones.

Causality in the PLA

It is no longer necessary to suppose as if the system "knows", in advance, which of its histories possesses the minimal action.

- ✓ The system does not need to "choose" history.
- ✓ The system does not need to "calculate" the action.
- ✓ Rather, it merely moves simultaneously along all possible histories.
- ✓ The combination of the possible histories creates the actual one.

Leibniz could say: "The action is the physical measure of the history's essence".

Possibilities in the interpretations of QM

1. One of the possible quantum events or histories becomes actual as a result of their observation.
N.Bohr , V.Heisenberg, J.von Neumann, E.Wigner

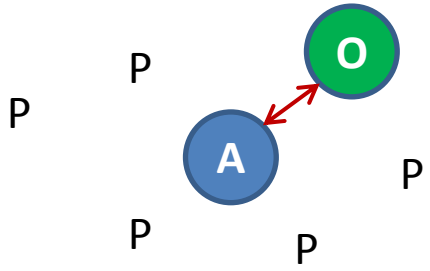
2. One of the possible quantum events or histories is merely detected by the measurement or by the interaction. Observation means are relative in relation to the objects.
V.Fock, Modal, Consistent histories, J.Wheeler, Existential

3. Each possible quantum event or history is realized as actual.
Many-Worlds

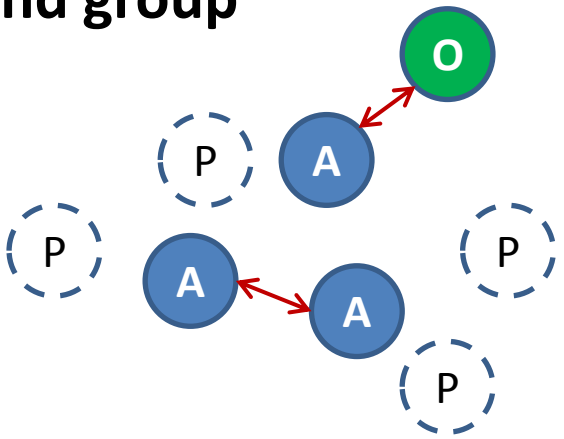
4. A certain set of possible quantum histories is realized at once. We observe the result of their combination.
E.Schrödinger, L.de Broglie, D.Bohm, R.Feynman

Four groups of the interpretations of QM

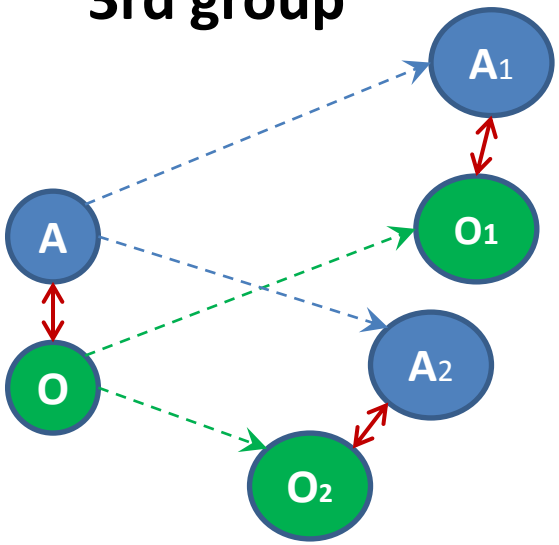
1st group



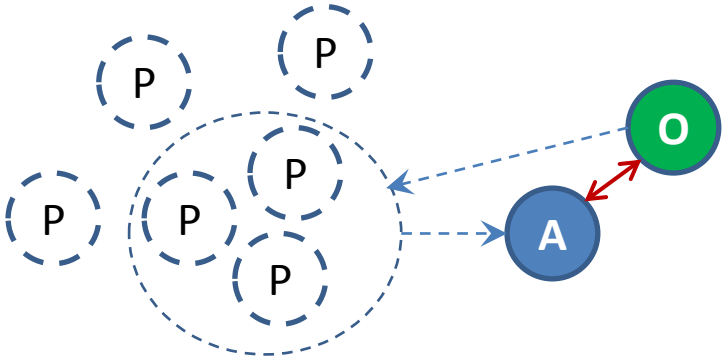
2nd group



3rd group



4th group



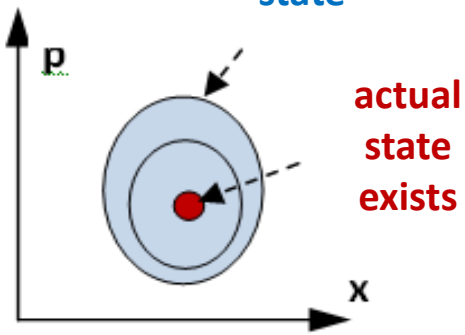
Part III

The Modal-Information approach

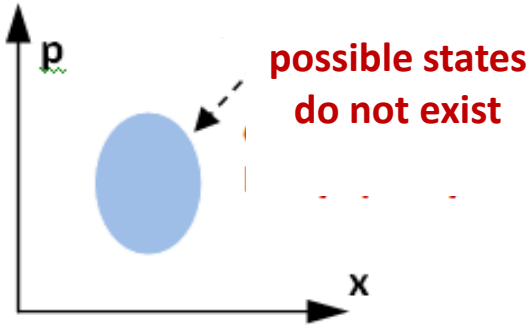
Three views on quantum probability

Classical realism	Anti-realism	Quantum realism
ψ (Psi)-epistemological interpretations		ψ (Psi)-ontic interpretations
Statistical, Local hidden variables	Some versions of Copenhagen, Quantum logic	Many-worlds, Bohmian, Informational, Qbism, Modal, Relational , Transactional, etc.

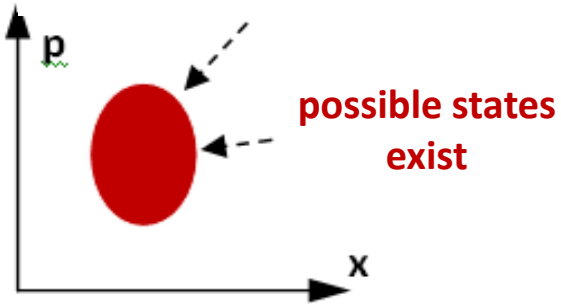
probability is incomplete knowledge of existing state



probability is knowledge about possible states after measurement



probability is information or knowledge about possible states existing prior to measurement



Quantum Information

One of the aims of the informational approach is the investigation of two concepts: the **knowledge** of the observer and **information**.

Two views on "information“:

- **The psi-epistemological approach:** the quantum information is the knowledge of the observer, which reduces uncertainty or degree of confidence a certain person.

- **The psi-ontic approach:** the quantum information is a separate entity, not always associated with a person.

Wheeler & Zurek

J. Wheeler:

- Information is at the origin of all **existence**.
- The being of the universe is a result of the **participations** of all observers in the process of the **exchange of information**.

W. Zurek:

- The information is **both** human knowledge and an **primary ontological entity**.
- The objective existence of the selected quantum states is acquired through the epistemological **information exchange with the environment.**

Quantum Darwinism

Zurek W.H. Quantum Darwinism. Nature Physics. 2009. Vol. 5(3), 181-188.

- ✓ Only states that produce **multiple informational imprints** on the environment can be found out.
- ✓ The origin of the emergent classicality is their ability of states to “procreate", to deposit **multiple records — copies of themselves** — throughout environment.
- ✓ Objective existence emerges from the quantum substrate as a consequence of **redundancy of information**.

The more information in the environment ,
the more existence.

From Dispositions to Information

- Every possible history of the system has **a disposition to be actualized** (*see the first attempt of summary*).
- Assume that the disposition is determined by **how much of information** may be extended in the environment .
- The amount of information spread in the environment depends on a **number of the near** possible histories.
- The **proximity** (compatibility) of the possible histories to each other is determined by the proximity of the **phase of probability amplitudes** of every possible history.

The more possible histories with close phases, the more information can spread in the environment.

The second attempt of summary

I. The more possible history's disposition to be actualized →
the more possible histories with close phases of the probability amplitudes.

II. The more possible histories with close phases →
the least action →
the more information spread in the environment.

III. The more information in the environment →
the more existence.

Three points

1. The Principle of Least Action (PLA) is a classical limit of Feynman Path Integral.

2. Leibniz could “explain” the meaning of the action. The modal interpretation of the PLA.

3. The action is related to redundancy of information in the environment (Quantum Darwinism).

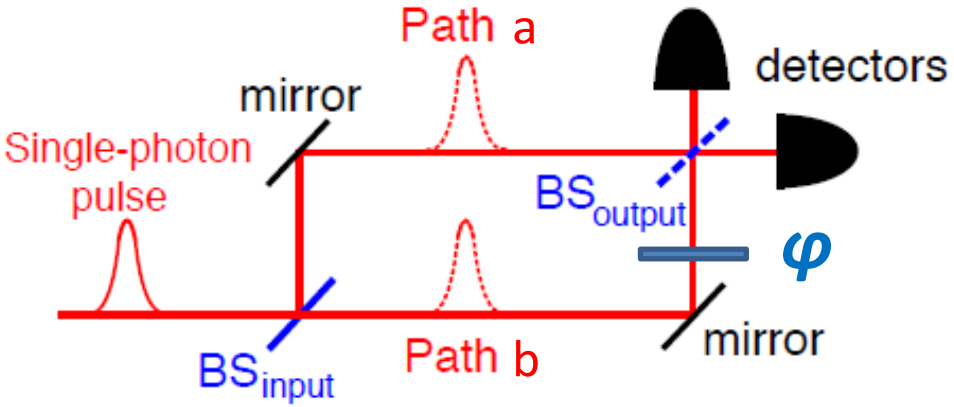
www.quantum-philosophy.com

www.vtpapers.ru

Appendix I

The experiment

The Mach-Zehnder interferometer



The second beam-splitter can be installed or removed at any time.

- If the second beam-splitter is installed, a photon takes two possible paths (in a superposition) with different polarizations.
- **Knowing the polarization we can say by which path the photon went.**

V. Jacques et al. (2007) Experimental realization of Wheeler's delayed choice experiment, Science, 315, 966.

Delayed choice experiment

A. The second beam-splitter is removed.

The detectors allow to determine which path has been followed by the photon. Paths do not interfere.

The photon behaves as a particle.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\mathrm{b}\rangle + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\varphi}\,|\mathrm{a}\rangle).$$

$$p_{\mathrm{a}} = p_{\mathrm{b}} = \tfrac{1}{2}.$$

$$p = p_a + p_b = |\psi_a|^2 + |\psi_b|^2$$

B. The second beam-splitter is installed .

The probabilities at the detectors depend on the phase-shift between two interfering paths.

The photon behaves as a wave.

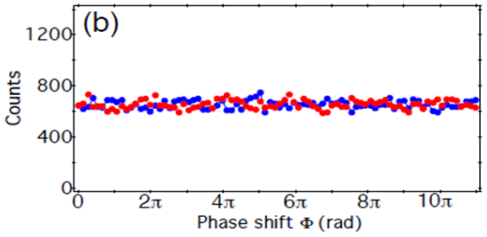
$$|\psi'\rangle = \cos\tfrac{\varphi}{2}\,|\mathrm{a}'\rangle - \sin\tfrac{\varphi}{2}\,|\mathrm{b}'\rangle.$$

$$p_{\mathrm{a}'} = \cos^2\tfrac{\varphi}{2} \text{ and } p_{\mathrm{b}'} = \sin^2\tfrac{\varphi}{2}.$$

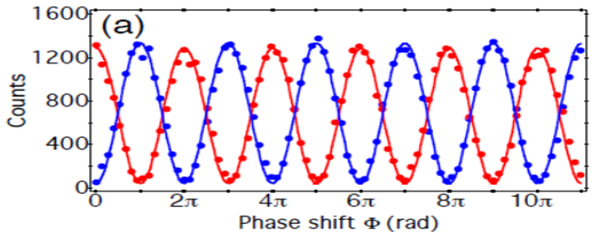
$$p = |\psi_a + \psi_b|^2$$

The Modal-Information approach

Photon = «particle»



Photon = «wave»



QM: the second beam-splitter changes the rule of summing of probability amplitude

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\mathrm{b}\rangle + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\varphi}\,|\mathrm{a}\rangle).$$

$$p_\mathrm{a} = p_\mathrm{b} = \tfrac{1}{2}.$$

$$|\psi'\rangle = \cos\tfrac{\varphi}{2}\,|\mathrm{a}'\rangle - \sin\tfrac{\varphi}{2}\,|\mathrm{b}'\rangle.$$

$$p_{\mathrm{a}'} = \cos^2\tfrac{\varphi}{2} \text{ and } p_{\mathrm{b}'} = \sin^2\tfrac{\varphi}{2}.$$

INFORMATION: the second beam-splitter changes and the phase-shift change the amount of information spread in the system "interferometer – photon – detectors - observer"

$I_\mathrm{a} = I_\mathrm{b} >> I_\mathrm{ab}$ $\quad I_\mathrm{ab} \to 0$ $\quad I_\mathrm{a} < I_\mathrm{ab} < I_\mathrm{b}$ $\quad 0 < I_\mathrm{a} < \max$ $\quad 0 < I_\mathrm{b} < \max$

ONTOLOGY: the second beam-splitter changes and the phase-shift change the dispositions to be actualized for the path *a* and the path *b*. The number of other possible paths close to path *a* or path *b* changes. The path *a* became compatible with the path *b* .

$D_\mathrm{a} = D_\mathrm{b} >> D_\mathrm{ab}$ $\quad D_\mathrm{ab} \to 0$ $\quad D_\mathrm{a} < D_\mathrm{ab} < D_\mathrm{b}$ $\quad 0 < D_\mathrm{a} < \max$ $\quad 0 < D_\mathrm{b} < \max$

N_pp near a $>> N_\mathrm{pp}$ near b (vice versa) $\quad N_\mathrm{pp}$ near a $< N_\mathrm{pp}$ near ab $< N_\mathrm{pp}$ near b

Appendix II

The possible histories in the interpretation of QM

Four approaches

Possibilities in the interpretations of QM

The possible events and possible histories can be used for the four groups of the interpretation of QM.

1. One of the possible quantum events or histories becomes actual as a result of their observation. Copenhagen.

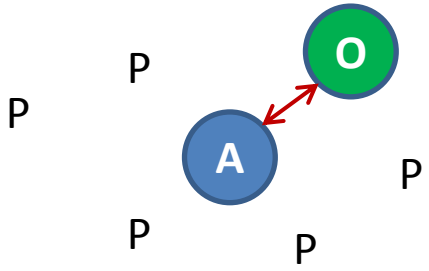
2. One of the possible quantum events or histories is merely detected by the measurement or by the interaction. Observation means are relative in relation to the objects. Fock, Modal, Consistent histories , Wheeler, Existential.

3. Each possible quantum event or history is realized as actual. Many-Worlds.

4. A certain set of possible quantum histories is realized at once. We observe the result of their combination. Schrödinger, De Broglie, Bohm, Feynman.

Four groups of interpretations of QM

1st group



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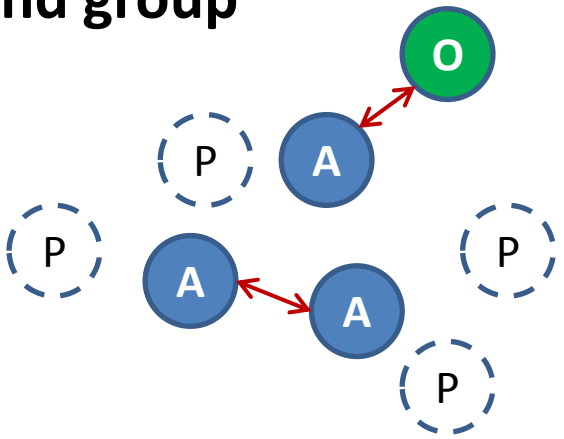

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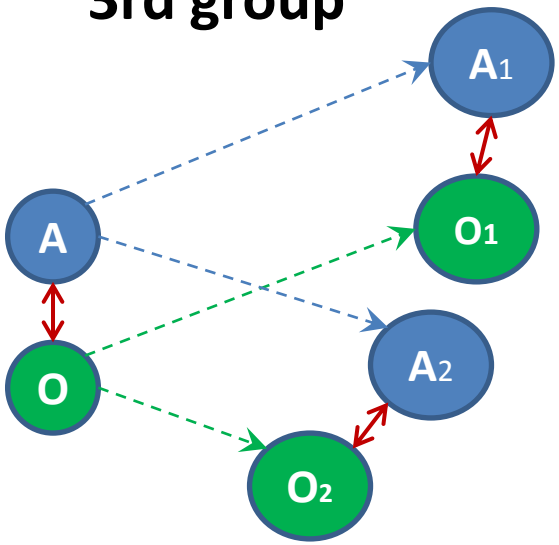
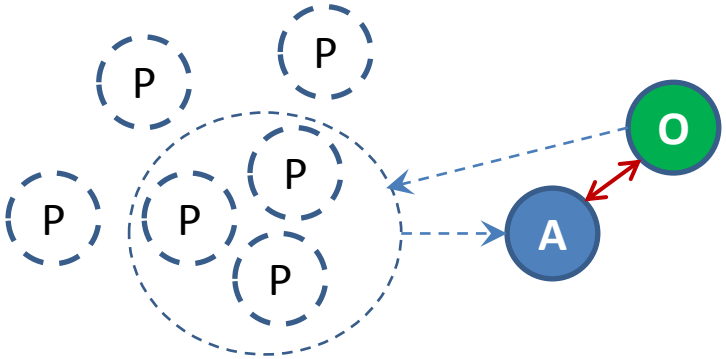


4th group



The first group of interpretations of QM

Bohr's version of Copenhagen interpretation. The only actual world is created by the measurement, and the "collapse of a wave function" does not describe a change the reality, but a change of our knowledge of the reality.

Heisenberg believed that mathematical laws of quantum theory can be considered in Aristotelian notions of "*dynamis*" or "*potency*", and that a notion of "possibility" occupies an intermediate position between objective material reality and subjective reality.

von Neumann and **Wigner** believed that the consciousness of the observer, which is connected with devices, creates a reality, destroying the superposition of possible states.

The second group of interpretations of QM

In contrast to Heisenberg, **Fock** believed that the state, which described by the wave function is objective. It is characteristic of the potential possibilities of one or another result of the interaction between the object and a device.

Oge Bohr (son). Quantum reality is described by operators rather than numbers. Quantum reality contains the whole spectrum at once real worlds.

Van Fraassen. A *dynamical state* determines the system's possible physical properties and their probabilities. A *value state* represents actual physical properties.

The measurement as well as any physical interaction randomly detects (but does not create, as Heisenberg supposed) one of the possible value states and makes it actual.

The second group of interpretations of QM

Modal interpretations (Bene and Dieks). Modalities are mere convenient tools for the description of the actual world and do not have their existence.

Modal-Hamiltonian interpretation (Lombardi and Castagnino) Quantum systems are within the realm of possibility that is not less real than the realm of actuality. The propensities follow a deterministic evolution independently of which possible facts become actual.

Consistent Histories (Gell-Mann and Hartle) select from all alternative quantum histories (Feynman paths) a set of coarse-grained coherent histories. Due to decoherence or "entanglement with the environment" only part of them interfere with each other. The reality before decoherence does not depend on the measurement.

The second group of interpretations of QM

Wheeler. The being of the whole universe is a result of the participation of an observer in the process of self-realization of the universe, through the exchange of information.

Existential interpretation (Zurek) based on a mechanism of decoherence with the environment. The ontological features of the actual states are selected only when the superposition principle is "turned off" by environment induced decoherence.

The objective existence of the selected states is acquired through the epistemological information exchange with the environment. This exchange of information exists objectively. It is supposed that information is not only human knowledge but the primary entity.

The third group of interpretations of QM

Many-Worlds interpretation of QM. Any measurement of quantum particles divides them into many copies. Each of copies actually exists in a parallel projection of a multiverse. Multiverse is composed of a quantum superposition of all its own possible branches or quantum worlds.

Any copy evolves according to the Schrödinger's equation, and a wave function is an ontological entity.

Everett denied any direct analogy with the transition the possibility to actuality that adopted in possibilism.

The fourth group of interpretations of QM

Schrödinger explained the actual trajectory by a set or a field of all possible trajectories. In an infinite number of possible trajectories none of these has the advantage to be implemented in a particular case, all these are equally real.

Instead of the implementation of only one possible entangled state (Heisenberg), all ones are summed up. It occurs due to resonance or interference of the waves.

De Broglie-Bohm. Non-local in space-time field of quantum potentials objectively exists independently of consciousness as a set of features. This non-local field depends on positions of all particles that at once influence the actual trajectory of the particle.

The fourth group of interpretations of QM

Feynman path integral. The photons do follow along all possible paths, and the summation of their probability amplitudes is not empty play in mathematics.

In our classical world, these possible histories are mutually-exclusive, although, at the quantum level these possible histories coexist in quantum superposition.