Nash-2 Equilibrium Concept: from Strict Competition to Tacit Collusion

Marina Sandomirskaia

CMSSE HSE

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Why to seek an extension of Nash equilibrium concept?

Classical Nash equilibrium theory faces sometimes difficulties in widely known economic models:

- It does not always exist in a number of games widely used in economics:
  - Price game in the Hotelling linear city model
  - Tullock contest
- It leads to "inadequate" game situation.
  - Prisoner's dilemma
  - Bertrand paradox
  - Hotelling minimum differentiation principle

How to overcome these problems?
One can change (complicate) the models, or revise the concept of rationality underlying the agents' behaviour.

Theoretical causes

Simon (1976):

"The choice that would be substantively rational for each actor depends on the choices made by the other actors; none can choose without making assumptions about how others will choose."

Nash equilibrium: myopic vs. sophisticated

Some experiments (Goeree, Holt, 2001; Camerer, Ho, Chong, 2004) demonstrate systematic deviations from Nash predictions.

Special discussion in JEL on *the Role of Bounded Rationality versus Behavioral Optimization in Economic Models* (Vol. 51 No. 2, June 2013)

Related concepts

- Smart$_n$ players (Stahl, 1993)
- Cognitive hierarchy (Camerer, Ho, Chong, 2004), or k-level rationality (Crawford at al., 2013)
- The largest consistent set (Chwe, 1994)
- Farsighted pre-equilibrium (Jamroga, Melissen, 2011)
- Theory of moves (Brams, Mattli, 1992)

2-stage predictions:

- Equilibrium in secure strategies (Iskakov M., Iskakov A., 2005)
- Perfect cooperative equilibrium (Halpern, Rong, 2010)
- Nash-2 equilibrium (Sandomirskaia, 2014)
  = Equilibrium contained by counter-threats (Iskakov M., Iskakov A., 2014)
  = Sequentially stable set (Fraser, Hipel, 1994: for discrete games)

Definition of Nash-2 equilibrium

2-person non-cooperative game in the normal form (pure strategies)

$$G = (i \in \{1,2\}; s_i \in S_i; u_i : S_1 \times S_2 \to R).$$

Definition (profitable deviation)

A deviation s'_i of player i at profile $s = (s_i, s_{-i})$ is profitable if

$$u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i}).$$

Definition (secure deviation)

A deviation s'_i of player i at profile $s = (s_i, s_{-i})$ is secure if for any profitable deviation s'_{-i} of the opponent at intermediate profile (s'_i, s_{-i}) player i is not worse off:

$$u_i(s'_i, s'_{-i}) \geq u_i(s_i, s_{-i}).$$

Definition (NE-2)

A strategy profile is a Nash-2 equilibrium if no player has a profitable secure deviation.

Secure and risky profiles

Definition (threat)

A profitable deviation of player i is called a threat to player $-i$ if player $-i$ gains less than in initial profile.

Definition (secure profile)

A profile is called secure if no player poses threats to the opponent.

Definition (risky profile)

A profile is called risky if there is at least one threat from one player to another.

The set of NE-2 is divided into two subsets:

- secure profiles (Equilibrium in Secure Strategies: Iskakov, 2005)
- risky outcomes (NE-2 $\setminus$ EinSS)

Interpretation

Secure part can be regarded as a *tough competition*:
agents protect themselves against any possible threats, even non-credible.

In **risky** situations agents have opportunities to harm one to another, but they do not actualize these threats as they are not credible.
Interpreted as *tacit collusion*.

Indeed, if explicit collusion is a NE-2, then it is in NE-2 $\setminus \mathrm{EinSS}$.

Theorem

If a collusion outcome is not a Nash equilibrium, then it is a risky profile.

Existence

Theorem

Nash-2 equilibrium in pure strategies exists in almost every finite game.

With some restriction in the definition of secure deviation the theorem holds for any continuous game with **bounded** utility function.

Important feature

In most cases Nash-2 equilibrium isn't unique. How to choose?

- EinSS or Nash equilibrium (dumping pricing in Hotelling model, Iskakov M., Iskakov A., 2013)
- Collusion (or Pareto efficient)
- Introducing a **measure of feasibility** on the set of NE-2 (Sandomirskaia, 2015)

Idea of measure building

Definition (secure path)

A path of profiles $\{(s_i^t, s_{-i}^t)\}_{t=1,\dots,T}$ is called a *secure path* if each its arc $(s_i^t, s_{-i}^t) \to (s_i^{t+1}, s_{-i}^{t+1}) = (s_i^{t+1}, s_{-i}^t)$ contains a secure profitable deviation s_i^{t+1} for some player i.

For any profile s denote the set of all NE-2 that can be reached from s through some secure path by NE-2_s.

The measure of feasibility on the set of NE-2

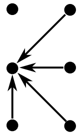
is calculated with the following rule:

$$\nu(s) = \frac{\mu(s)}{\mu(S_1 \times S_2)} + \sum_{\tilde{s}: s \in \text{NE-}2_{\tilde{s}}} \frac{\mu(\tilde{s})}{\mu(\text{NE-}2_{\tilde{s}})\mu(S_1 \times S_2)},$$

$\forall s \in$ NE-2, μ is a measure on the action set.

Example 1: finite game

	L	R
T	(2/3, 1/3)	(-1, 2)
C	(1/2, 1/2)	(1, 0)
B	(1, 0)	(0, 1)



(T,L) is an isolated NE-2, thus $\nu(T, L) = 1/6$.

$deg^{-}(C, L) = 4$. Thereby, $\nu(C, L) = \frac{1}{6}(1 + 4) = 5/6$.

Example 2: Bertrand duopoly with homogeneous product

- two firms producing a homogeneous product with equal marginal costs c;
- D demand is a linear function of the price $Q(p) = 1 - p$.

$$\pi_i(p_i, p_{-i}) = \begin{cases} (p_i - c)Q(p_i), & \textit{if } p_i < p_{-i}, \\ (p_i - c)Q(p_i)/2, & \textit{if } p_i = p_{-i}, \\ 0, & \textit{if } p_i > p_{-i}. \end{cases}$$

NE-2 provides any price level $p = p_1 = p_2 \in [c, 1]$.

In particular, monopoly price level $p = \frac{1+c}{2}$ is in NE-2.

There is a secure path from each profile (p_1, p_2), $p_1 \neq p_2$, $p_1, p_2 \in [c, 1]$, to NE-2 profile (p, p) with $p \in [c, \min(p_1, p_2)]$.

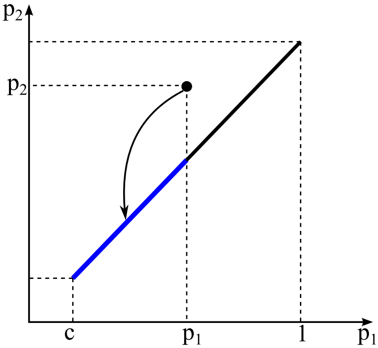


Fig. 1. The structure of secure paths in Bertrand model

The measure:

$$\nu(p,p) = \frac{2}{1-c}\left(\ln\frac{1-c}{p-c} - \frac{1-p}{1-c}\right), \quad \forall p \in [c,1].$$

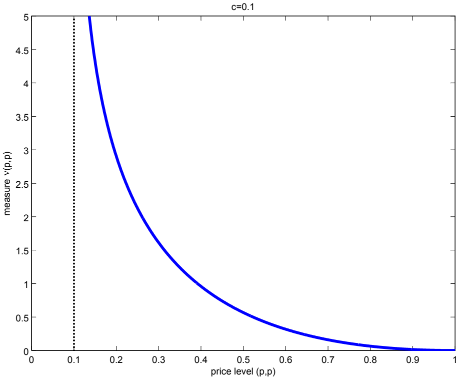


Fig. 2. The measure of feasibility on the set of NE-2 in Bertrand model with $c = 0.1$

Cournot duopoly

Two firms $i = 1, 2$ produce q_1, q_2 units of homogeneous product with equal constant marginal costs c per unit.

Equilibrium price is $\qquad p(Q) = 1 - Q$,
$Q = q_1 + q_2$ is total output.

i-th firm profit is
$\pi_i(q_1, q_2) = q_i \cdot (p(Q) - c) = q_i(1 - q_1 - q_2 - c)$.

Theorem

Nash-2 equilibria are profiles (q_1, q_2) from

a) secure set $\left\{ \left(b;\ \frac{1-c-b}{2}\right) \cup \left(\frac{1-c-b}{2};\ b\right) \mid \quad b \in \left[\frac{1-c}{3};\ 1-c\right) \right\}$,

b) risky set $q_1 = q_2 \in (0,\ (1-c)/3)$*, including collusive outcome* $(1-c)/4,\ (1-c)/4$.

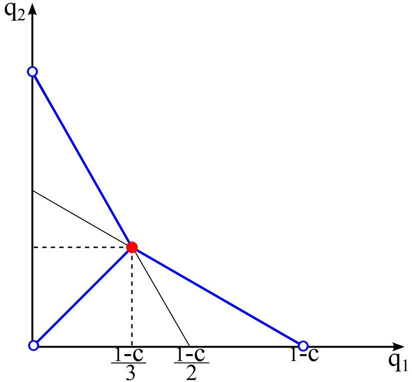


Fig.3. Red point is NE, NE-2. Blue lines are NE-2.

NE-2 set provides a number of regimes with various degree of toughness from competitive till collusive.
Oligopolistic equilibrium (d'Aspremont, Dos Santos Ferreira, Gerard-Varet, 2007).

Bertrand duopoly with differentiated products

- two firms producing imperfect substitutes with marginal costs equal c_1 and c_2, respectively;
- Firms' demand curves are:

$$q_1 = 1 - p_1 - \gamma(p_1 - p_2),$$

$$q_2 = 1 - p_2 - \gamma(p_2 - p_1).$$

The firms' profits are

$$\pi_1(p_1, p_2) = (p_1 - c_1)(1 - p_1 - \gamma(p_1 - p_2)).$$

$$\pi_2(p_1, p_2) = (p_2 - c_2)(1 - p_2 - \gamma(p_2 - p_1)).$$

$\gamma = 0$ – monopoly;
$\gamma \to \infty$ – homogeneous product.

Boundary NE-2: a closed-form solution

Non-negativity of markup and demand:

$$p_1 \geq c_1, \quad p_2 \geq c_2,$$

$$q_1(p_1, p_2) \geq 0, \quad q_2(p_1, p_2) \geq 0.$$

NE-2 prices exceed best response level:

$$p_1 \geq \frac{1 + \gamma p_2 + c_1(1+\gamma)}{2(1+\gamma)}, \quad p_2 \geq \frac{1 + \gamma p_1 + c_2(1+\gamma)}{2(1+\gamma)}.$$

At NE-2 firms get not less then their guaranteed gains:

$$\pi_1(p_1, p_2) \geq \frac{(1 - c_1(1+\gamma))^2}{4(1+\gamma)}, \quad \pi_2(p_1, p_2) \geq \frac{(1 - c_2(1+\gamma))^2}{4(1+\gamma)}.$$

The absence of secure profitable deviations:

$$\left(\frac{1-c_1}{2} - \frac{\gamma(1+\gamma)(p_2-c_2)}{2(1+2\gamma)}\right)\left(\frac{1+2\gamma+\gamma^2 c_2-(1+\gamma)^2 c_1}{2(1+\gamma)} + \frac{3}{2}(p_2 - c_2)\right) \leq \pi_1(p_1, p_2),$$

$$\left(\frac{1-c_2}{2} - \frac{\gamma(1+\gamma)(p_1-c_1)}{2(1+2\gamma)}\right)\left(\frac{1+2\gamma+\gamma^2 c_1-(1+\gamma)^2 c_2}{2(1+\gamma)} + \frac{3}{2}(p_1 - c_1)\right) \leq \pi_2(p_1, p_2).$$

Dynamic on γ

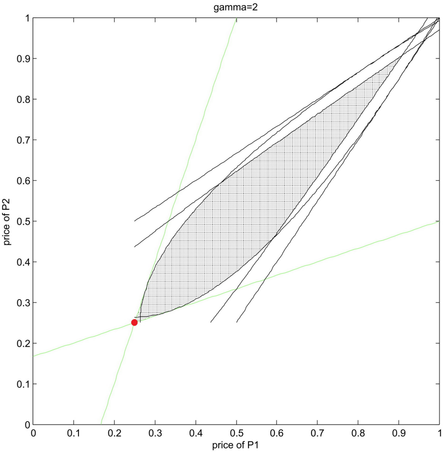


Fig. 4a. $c_1 = c_2 = 0$, $\gamma = 2$. Red point is NE, ESS, NE-2. Shaded area is NE-2.

Dynamic on γ

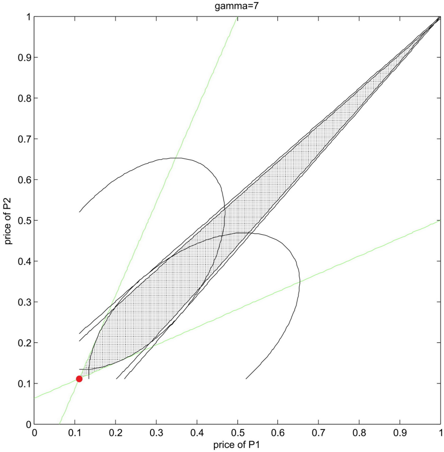


Fig.4b. $c_1 = c_2 = 0$, $\gamma = 7$. Red point is NE, ESS, NE-2. Shaded area is NE-2.

Dynamic on γ

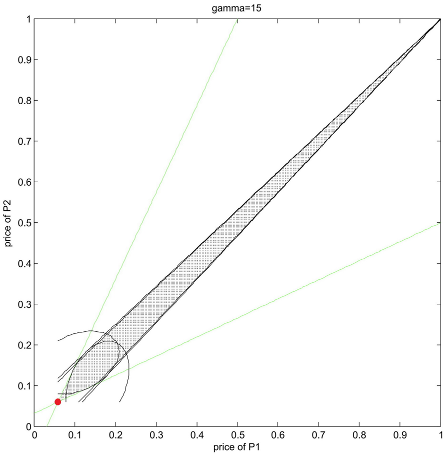


Fig.4c. $c_1 = c_2 = 0$, $\gamma = 15$. Red point is NE, ESS, NE-2. Shaded area is NE-2.

Dynamic on $c_1 - c_2$

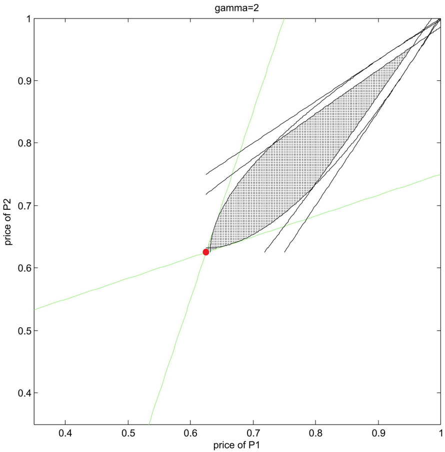


Fig.5a. $c_1 = c_2 = 0.5$, $\gamma = 2$. Red point is NE, ESS, NE-2. Shaded area is NE-2.

Dynamic on $c_1 - c_2$

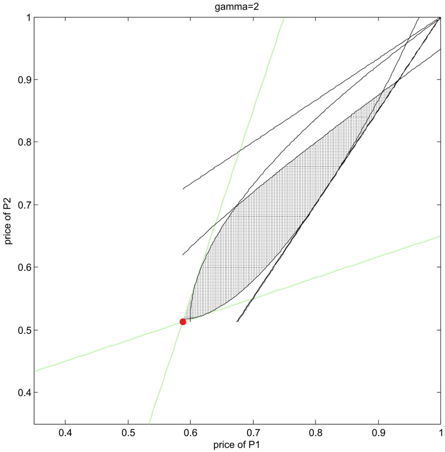


Fig.5b. $c_1 = 0.5$, $c_2 = 0.3$, $\gamma = 2$. Red point is NE, ESS, NE-2.
Shaded area is NE-2.

Dynamic on $c_1 - c_2$

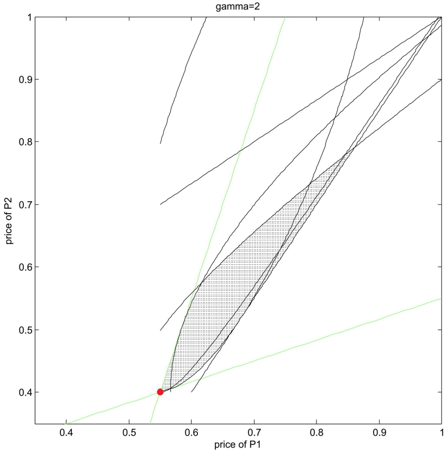


Fig.5c. $c_1 = 0.5$, $c_2 = 0.1$, $\gamma = 2$. Red point is NE, ESS, NE-2. Shaded area is NE-2.

Tullock contest (rent-seeking model, 1967)

The contest success function translates the effort x of the players into the probabilities that each player will obtain the resource R.

$$p_i(x_i, x_{-i}) = \frac{x_i^\alpha}{x_i^\alpha + x_{-i}^\alpha}, \quad x \neq 0, i = 1, 2.$$

If $x = 0$ then $p_i = p_{-i} = 1/2$.

The payoff function: $u_i(x_i, x_{-i}) = Rp_i(x_i, x_{-i}) - x_i$.
Without loss of generality assume $R = 1$, $x_i \in [0, 1]$.

When $\alpha > 2$ pure NE doesn't exist.

Secure NE-2 are found in (Iskakov M., Iskakov A., Zakharov, 2013)

Simulation results: efforts, $\alpha = 0.7$

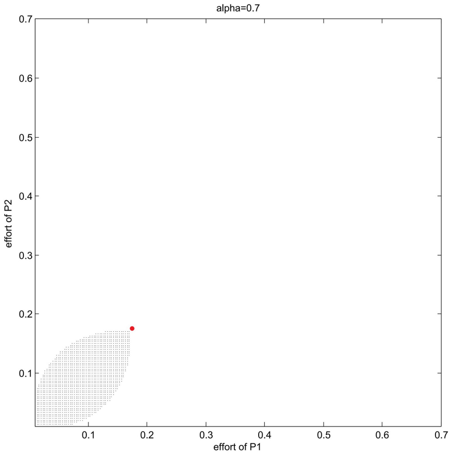


Fig.6a. Red point is NE, ESS, NE-2. Shaded area is NE-2.

Simulation results: efforts, $\alpha = 1.5$

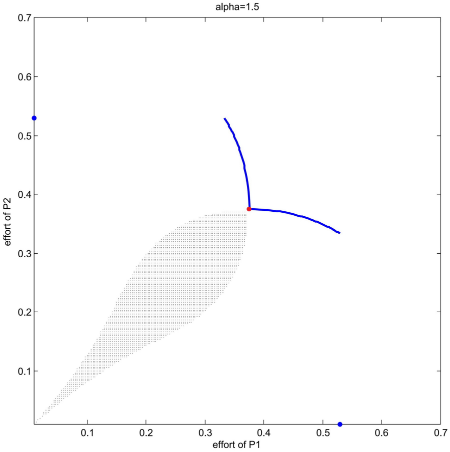


Fig.6b. Red point is NE, ESS, NE-2. Blue curve and points are ESS, NE-2. Shaded area is NE-2.

Simulation results: efforts, $\alpha = 2.3$

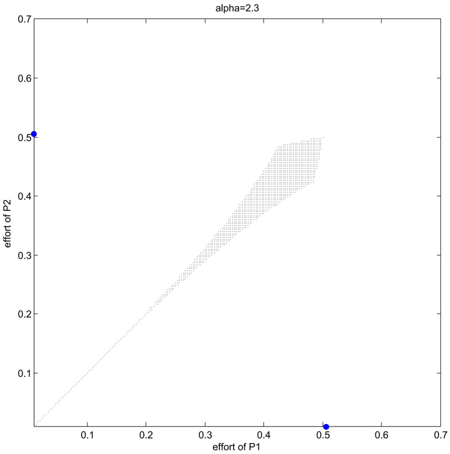


Fig.6c. Blue points are ESS, NE-2. Shaded area is NE-2.

Simulation results: PROFITS, $\alpha = 1.5$

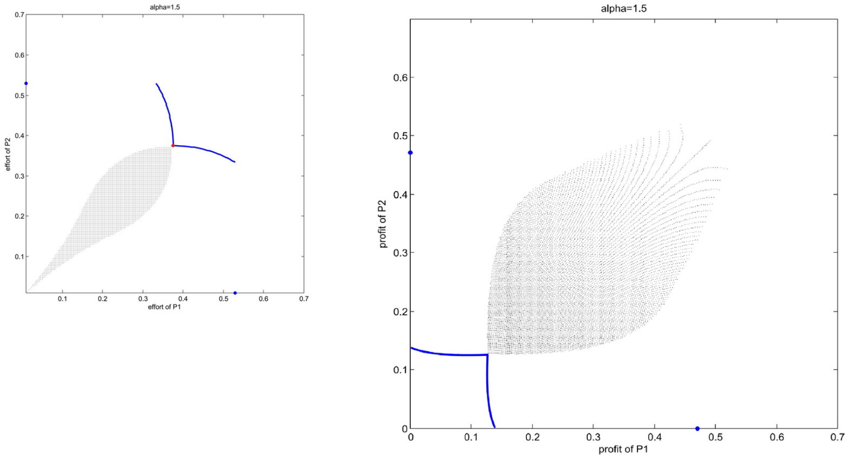


Fig.7. Curves and blue points are ESS and NE-2 payoffs, shaded area is set of profits at NE-2.

Efficiency

Rent dissipation is the ratio $(x_1 + x_2)/R$.

The higher is the degree of rent dissipation, the lower is the efficiency of the equilibrium.

For $\alpha > 2$ NE in mixed strategies is completely dissipated (Baye et. al., 1994).

All secure "non-monopolistic" NE-2 are less efficient than NE.

All risky NE-2 are more efficient!

Summarizing...

Additional example (closed-form solutions):

- Hotelling linear city model (2014)

Advantages of NE-2:

+ Existence
+ Strategic motivation for tacit collusion

Challenges of NE-2:

- Multiplicity
- Empirical support

Thank you for your attention!

E-mail: sandomirskaya_ms@mail.ru